Exam PPP

April 10, 2018

- Put your name and student number on each answer sheet.
- Answer all questions short and to the point, but complete; write legible.
- Final point grade = total number of points/10.

1. **Pions: isospin and decays (25 points)**
The pion is the lightest meson and it has a spin and parity of $J^P = 0^-$ (pseudoscalar). This meson is composed a light quark and antiquark, e.g. $q = (u, d)$. The isospin (I) of the light quark is $I = 1/2$ with $I_z = +1/2$ for the u quark and $I_z = -1/2$ for the d quark. The u quark has an electric charge of $+2/3e$ and the charge of d quark is $-1/3e$. The d quark (5 MeV) is slightly heavier than the u quark (2 MeV), but both much smaller than the mass of the pion itself (140 MeV). The isospin of the pion is one ($I = 1$).

   a) Rotations in isospin space ($u \leftrightarrow d$) are considered as a good symmetry for the strong interaction leading to the conservation of isospin. The mass differences between the three charged-pion states are very small as a consequence of isospin symmetry. Explain the origin of isospin symmetry and discuss possible sources that explicitly break the symmetry.
   Isospin symmetry follows from SU(3) color symmetry. The fact that the strong interaction, e.g. gluon exchange, is blind for the type of quark flavor and the fact that the up and down quarks have masses that are practically zero compared to the masses of hadrons, leads to an effective SU(2) flavor symmetry for q=(u,d) as basic representation. The symmetry is broken because the masses of the u and d quarks do slightly differ. Moreover, the electric charge differs as well, and hence, the e.m. interaction breaks the symmetry. Note: since the word "origin" could also be read as a historical origin, the answer could also be that Heisenberg made a proposal in which neutrons and protons were considered as identical particles if one switches of the e.m. interaction and ignore the small mass difference. Later this was traces back to the (u,d)-quark.

   b) Discuss the quark-flavor contents and the electric charges of the three z-isospin projected states ($I_z = -1, 0, +1$). Explain why the $I_z = 0$ state is slightly lighter (smaller in mass) than the $I_z = -1, +1$ states.
   $I_z = -1$ refers to $|d\bar{u}>$ state ($=\pi^-$), $I_z = 0$ refers to a linear combination if $|u\bar{u}>$ and $|d\bar{d}>$ ($=\pi^0$), and $I_z = +1$ corresponds to $|u\bar{d}>$ ($=\pi^+$). The charged pions are slightly heavier because their self-energy is larger due to their electric fields, which is absent for the π^0.

   c) Which orbital angular momenta are allowed for a $J^P = 0^-$ state composed of a $q\bar{q}$ pair? Can you form an isospin $I = 0$ state with $J^P = 0^-$ and $q = (u, d)$? Motivate your answers.
   The intrinsic parities of the q and $\bar{q}$ are opposite in sign. Moreover, the orbital angular momentum between the quark pairs gives a factor $(-1)^L$, hence the parity $P = (-1)^{L+1}$ for a quark antiquark pair. To conserve parity, L can only be even (0,2, etc.). The total angular momentum is the quantum mechanical sum of L and the two intrinsic spins of the two quarks. Since $L = 1$ is excluded by parity, the only possible allowed configuration will be that the two intrinsic spins add up to $S = 0$ (singlet spin state) in combination with

$L = 0$. Hence, only $L = 0$ is possible. Yes, one can have an $I = 0$ pseudoscalar meson. The lightest one is called the η.

d) The lifetime of the $I_z = 0$ pion is significantly shorter than of the $I_z = -1, +1$ pions. Explain this qualitatively.
The charged pions can only decay with the weak interaction, via an intermediate W boson. The π^0 can also decay electromagnetically via the emission of two photons. The e.m. force is much stronger than the weak force, and hence, the π^0 will have a much shorter lifetime.

e) The $I_z = -1, +1$ pions decay predominantly into a (anti)muon (μ) and (anti)neutrino pair (ν). The decay to an (anti)electron and (anti)neutrino is heavily suppressed compared to the muon decay mode. Explain the origin of this difference. Please note that the muon has a mass of about 100 MeV, whereas the electron has a mass of 0.5 MeV.
The (anti)muon/electron and (anti) neutrino will decay back-to-back in the center-of-mass of the pion, hence, their momentum direction will be opposite. The weak interaction only couples to left(right)-handed chirality components of the (anti)leptons. The neutrino is practically massless, which means that helicity is equivalent to chirality. For simplicity lets consider the neutrino in combination with the e^+/μ^+ (similar discussion for the opposite C case). The spin direction of the neutrino will be opposite to its momentum direction (left-handed helicity). Since the pion has no spin and angular momentum is conserved, the e^+/μ^+ helicity will be left-handed (LH). The mass of the positron is very small compared to its momentum, and hence the RH chiral component will be tiny. For the LH-helicity μ^+, the RH chirality component is still sizeable because of its relatively large mass. Therefore, the muon decay mode will be significantly larger than the positron decay.

2. **The Higgs phenomena (25 points)**
The Standard Model (SM) of particle physics includes a spinless boson called the *Higgs*. This particle was predicted by theory and, later, discovered experimentally in 2012 at LHC. The *Higgs mechanism* is related to two phenomena, namely *spontaneous symmetry breaking* (SSB) and *Gauge invariance*. As a reminder, the Lagrangian density for a complex scalar field, ϕ, is given by $L = \frac{1}{2} D_\mu \phi^* D^\mu \phi - V(\phi^* \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$. D represents the covariant derivative $D_\mu = \partial_\mu + iA_\mu$ whereby A_μ represents a vector field. The dynamics of the field A_μ is described by the field tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Please note that this Lagrangian corresponds to a simplification of the actual situation. It represents, the dynamics of a charged boson field, ϕ, interacting with an electromagnetic photon field, A_μ. In the SM, it is actually the SU(2)-weak symmetry that undergoes a spontaneous symmetry breaking, whereby ϕ represents the Higgs field and A the Gauge bosons of the weak interaction $(W^\pm, Z)$.

a) Discuss qualitatively the concept of SSB, how this gives rise to the so-called *Goldstone boson* and *Higgs quanta*. How can one see from the expression of L that the symmetry is spontaneously broken?
The SSB can be identified via the potential term $V(\phi\phi^*)$. Take a potential of the form $V = \mu\phi\phi^* + \lambda(\phi\phi^*)^2$. Such a potential will be symmetric in U(1), e.g. if you make a rotation in the complex plane: $\phi \to e^{i\theta}\phi$. In the case both μ and λ are positive, you get a potential that looks like a bowl with a minimum at $\phi = \phi^* = 0$. In that case, you get a single ground state with expectation value of the field that is zero. For a negative μ and a positive λ, the potential will look like a mexican hat: still U(1) symmetric, however, with many possible ground states in the minimum of the hat. This would be equivalent to the SSB situation. The field ϕ can be decomposed in a state that corresponds to oscillations in the direction at which the potential stays constant in the angular direction, and one in the radial direction which corresponds to a field with of potential that is in first order quadratic. The first field corresponds to the Goldstone boson, e.g. massless quanta and the second to a field with a massive quanta that can be called the Higgs. A more appropriate representation of the field in the case of SSB would be $\phi = \rho e^{i\alpha}$, whereby α is the field in angular direction, e.g. the Goldstone boson, and ρ in radial direction, e.g. the Higgs field.

b) The Lagrangian L as described above has a local U(1) symmetry (=Gauge invariance). This implies that the transformation $\phi \to e^{i\theta}\phi$ and $A_\mu \to A_\mu - \partial_\mu\theta$ will leave L invariant. Show that L is indeed Gauge invariant.
To see that the new Lagrangian is Gauge invariant, we first evaluate the covariant derivative of the transformed fields $\phi' = \phi e^{i\theta(x)}$:

$$
\begin{aligned}
D\phi' &= (\partial\phi + i\phi\partial\theta)e^{i\theta} + i(A - \partial\theta)\phi e^{i\theta} \\
&= (\partial + iA)\phi e^{i\theta} = D\phi e^{i\theta}.
\end{aligned}
$$

Similar expression of $D\phi'^*$ with term $e^{-i\theta}$, which leads to $D\phi' D\phi'^* = D\phi D\phi^*$, e.g. Gauge invariant. Also the potential term is Gauge invariant simply since it only depends on $\phi\phi^*$, and hence, the transformed version will get a factor $e^{i\theta}e^{-i\theta} = 1$, therefore, $\phi'\phi'^* = \phi\phi^*$. The field tensor, which only contains derivatives, is also Gauge invariant since

$$
F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu = \partial_\mu(A_\nu - \partial_\nu\theta) - \partial_\nu(A_\mu - \partial_\mu\theta) = \partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu},
$$

since $\partial_\mu\partial_\nu\theta = \partial_\nu\partial_\mu\theta$. Hence L is Gauge invariant.

c) In the case, there is no spontaneous symmetry breaking, the mass of the field A_μ will be zero. How would a mass term look like in L and why is it not allowed to explicitly add such a term to L.

A mass term looks like $\frac{1}{2}m^2A^2$, e.g. a term that is proportional to the square of the field. Such a term is clearly not Gauge invariant and can, therefore, not be added explicitly in the Lagrangian.

d) SSB will lead to an effective mass for A_μ. Demonstrate this by analyzing L in the case of SSB. What happens to the Goldstone boson? Motivate your answers.
In the case of SSB, the expectation value of the field ϕ will be shifted away from the origin, say by a real value of f with units of energy. To first approximation, we can state that $\phi = fe^{i\alpha}$. The term with covariant derivatives will become

$$\frac{1}{2}D\phi D\phi^* = \frac{1}{2}(\partial + iA)fe^{i\alpha}(\partial - iA)fe^{-i\alpha} = \frac{1}{2}(\partial\alpha + A)^2 f^2.$$

Note that we can always make a Gauge transformation $A \to A - \partial\alpha$ which would leave L invariant. Hence, we can simplify the equation above to a term that summarizes to $\frac{1}{2}f^2A^2$. The Goldstone field α can be removed by a transformation, which means it is not physical quantity, and we obtain an effective mass term for the field A with a mass that is equal to the shift of the vacuum expectation value, f. In summary, A gets a mass and the Goldstone boson disappears.

e) The Higgs phenomena as described above explains how Gauge bosons obtain a mass. The Higgs field is also responsible for the mass of fermions in the SM. Qualitative explain how the fermions obtain a mass.
The reason that fermions in the SM obtain a mass is directly related to the breaking of parity of the weak interaction. Only left-handed fields are allowed to participate in the interaction, which means that only left-handed fermions have a weak charge, and right-handed fermions do not have a weak charge. In the U(1) equivalent, it would imply that only left-handed fermions have an electric charge. If one works out the Dirac equation in terms of left and right-handed components, one obtains an expression similar to

$$i\left(\frac{\partial\Psi_R}{\partial t} + \alpha_i\frac{\partial\Psi_R}{\partial x^i}\right) = m\Psi_L,$$
$$i\left(\frac{\partial\Psi_L}{\partial t} - \alpha_i\frac{\partial\Psi_L}{\partial x^i}\right) = m\Psi_R,$$

with α_i representing three of the four Dirac matrices. Note that in a particular representation, the fourth Dirac matrix, often called β, interchanges left and right-handed components. This gives rise to two coupled equations, each with both left- and right-handed fields, as illustrated in the above equations. U(1) transformation implies the following rules: $\Psi_L \to e^{i\theta}\Psi_L$ and $\Psi_R \to \Psi_R$. This since right-handed fields have no charge and left-handed fields do. Clearly, the above equations are not invariant under U(1), and hence, charge is not conserved. To solve this, one can replace m by the interaction with the field, ϕ, with a term like $m = g\phi^*$ and $m = g\phi$ for the right-hand side of the upper and lower equations above, respectively. The charged field ϕ would transform like $\phi \to e^{i\theta}\phi$ and the above equations become symmetric again under U(1). g is called the Yukawa coupling constant, and it reflects the coupling strength of the Higgs field with the fermions. In the case of SSB, one can approximate ϕ by f (the Goldstone boson field is gone), and hence the masses of the fermions become gf. Since f is non-zero, the fermions obtain a mass.

3. **Kaon oscillations (25 points)**
Kaons are mesons that contain one strange (anti)quark combined with a light up/down (anti)quark. Their spin-parity is the same as for a pion, namely $J^P = 0^-$. In this problem, we focus on the electrically neutral kaons, namely K^0 ($\bar{s}d$) and $\bar{K}^0$ ($s\bar{d}$). Using a photon beam impinging on a proton target at rest, one can produce these mesons in the reaction

$$\gamma + p \to K^0 + \Sigma^+,$$

whereby Σ^+ represents a baryon with quark configuration uus (hyperon). The Σ baryon can be identified by studying its decay into a nucleon (proton or neutron) and a pion (π^0 or π^+). The masses of the proton, K^0, Σ^+, and π are 938 MeV, 497 MeV, 1189 MeV, and 140 MeV, respectively.

a) Calculate the minimum energy of the photon beam that would be needed to produce kaons according to the reaction given above. At this energy, would it be possible to produce as well $\bar{K}^0$? Motivate your answer.
In the center-of-mass the minimum energy corresponds to the sum of the mass of sigma and kaon. This one can relate to the lab. by making use of the Lorentz invariant expression:

$$(E^2_{\rm tot} - p^2_{\rm tot})_{\rm lab} = (E^2_{\rm tot})_{\rm cm}.$$

Note that $p_{\rm tot}$ in the center-of-mass is zero. Hence,

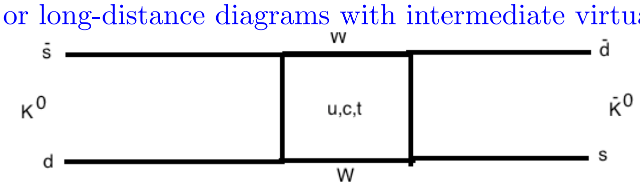
$$(E_\gamma + m_p)^2 - E_\gamma^2 = (m_K + m_\Sigma)^2,$$

which leads to

$$E_\gamma = \frac{(m_K + m_\Sigma)^2 - m_p^2}{2m_p},$$

which gives a threshold energy E_γ=1046 MeV. In order to produce a $\bar{K}^0$, one should consider a reaction that preserves upness, downness, and strangeness. The reaction with the smallest threshold for this, would be $\gamma + p \to K^0 + \bar{K}^0 + p$. The threshold for this reaction is E_γ=1520 MeV which can be obtained by replacing $m_K + m_\Sigma$ by $2m_K + m_p$. Hence, if the photon energy is below this value, it will not possible to produce a $\bar{K}^0$ in the reaction.

b) The K^0 and $\bar{K}^0$ will mix, leading to oscillations. What are the underlying mechanisms on the quark level that give rise to mixing? Make use of Feynman diagrams to illustrate your answer.
A typical mechanism that would mix kaons is the so-called flavor-changing neutral currents or long-distance diagrams with intermediate virtual pion pairs.



c) Express the mass eigenstates of the neutral-kaon system in terms of the flavor eigenstates (K^0 and $\bar{K}^0$) under the assumption that CP is a good symmetry.
The K^0 and $\bar{K}^0$ are not CP eigenstates, since $CP|K^0 >= -|\bar{K}^0 >$ and $CP|\bar{K}^0 >= -|K^0 >$. The quantum superpositions of K^0 and $\bar{K}^0$ can give CP eigenstates, namely

$$|K_1 >= (|K^0 > -|\bar{K}^0 >)/\sqrt{2},$$
$$|K_2 >= (|K^0 > +|\bar{K}^0 >)/\sqrt{2},$$

where K_1 is the CP even state and K_2 the CP odd state. These are the mass eigenstates of the neutral kaon system.

d) One of the mass eigenstates, named K_S, decays predominantly into a pair of pions with $c\tau = 2.7$ cm (τ is its lifetime and c is the speed of light). Which orbital angular momenta of the pion pair are allowed in such decay. Argue that parity must be violated in this reaction. What about isospin symmetry? Motivate your answers.

The parity of a two-pion system is given by $P(\pi) \times P(\pi) \times (-1)^L$ with L the orbital angular momentum between the two pions and $P(\pi)$ the intrinsic parity of the pion. Hence, the parity is given by $(-1)^L$. Since the kaon has no spin, the total angular momentum of the two-pion system should be zero in order to conserve angular momentum. Since, the pions have no spin, $L = 0$, corresponding to an even parity. The parity of the kaon is odd, hence, parity is violated. Note that the weak interaction does not preserve parity. C-parity is violated as well since CP is conserved and P is violated. Note that K_S corresponds to the K_1 mass eigenstate. The isospin of a kaon is 1/2, since it is composed of one light quark. The isospin of the two-pion system is either 0,1,2. Therefore, isospin is violated as well in this weak interaction.

e) The other mass eigenstate is called K_L, with $c\tau = 15.3$ m. This particle decays predominantly into three pions. What is the reason why the K_L hardly decays into two pions? Why has the K_L a much larger $c\tau$ than the K_S? Motivate your answers.

The K_L is the odd CP eigenstate. Assuming CP conservation (which is only slightly broken in the weak interaction), it cannot decay into two pions, since the latter is CP even. The Q-value for the $K_L \to 3\pi$ is much smaller than $K_S \to 2\pi$. As a consequence, the phase space for the K_L is very small, which makes its decay rate much smaller than the K_S decay. The decay rate is inversely proportional to the lifetime. Therefore, the lifetime of the K_L is much larger than that of the K_S.

4. **Electron scattering (25 points)**
The structure of the proton can be studied via the elastic electron-scattering process $e^- + p \rightarrow e^- + p$. For this, an electron beam hits a hydrogen target. The differential cross section in natural units of this process can be expressed as

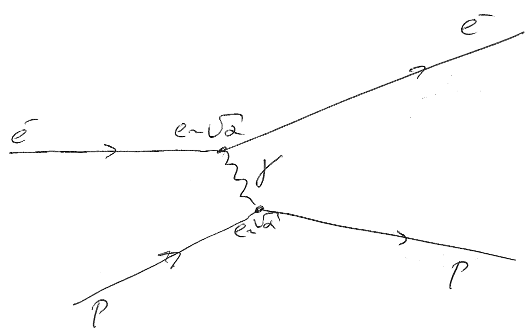
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \left(\frac{E'}{E}\right) \left(G_1(Q^2)\cos^2(\theta/2) + 2\tau G_2(Q^2)\sin^2(\theta/2)\right),$$

with $\tau = Q^2/(4M^2)$ and G_1, G_2 are structure functions that represent the squares of the electric and magnetic form factors of the proton. The fine-structure constant $\alpha = e^2/(4\pi) = 1/137$ and M refers to the mass of the proton (938 MeV). The parameter $Q^2 = -q^2$ relates to the square of the four-momentum transfer $q = k - k'$ whereby k, k' refers to the four-momentum of the initial and final-state electron. The incoming electron energy is indicated as E and the energy of the scattered electron is E'. These energies are related to each other via the recoil formula

$$E' = \frac{E}{1 + \frac{E}{M}(1 - \cos\theta)}.$$

The angle θ refers to the scattering angle of the electron in the laboratory frame. You may ignore the mass of the electron.

a) Make a sketch of the Feynman diagram of the electron scattering process and use this to motivate why the cross section scales like α^2.



Since the amplitude scales with the multiplication of the charges of the two vertices, it will scale with factor α. The cross section scales as the square of the amplitude, and hence, scales as α^2.

b) Demonstrate or argue that Q^2 (q^2) is always positive (negative) in electron scattering.
Working out the expression q^2, leads to

$$\begin{aligned} q^2 &= (k - k')^2 = k^2 + k'^2 - 2kk' = 2m_e^2 - 2(EE' - \vec{p} \cdot \vec{p}') \\ &\approx -2(EE' - EE' \cos\theta) = -2EE'(1 - \cos\theta) < 0, \end{aligned}$$

where we ignored the mass of the electron. Note that E, E', and $(1 - \cos\theta)$ are always positive, thereby, q^2 is negative and Q^2 positive.

c) The differential cross section as presented in the equation above reveals a divergence when θ approaches zero (which corresponds to Q^2 approaching zero). What is the physical interpretation of this divergence and argue why in practice the cross section will not diverge.
The divergence originates from the infinite range of the electromagnetic force. The photon is massless and, hence, the propagator term will be like $1/q^2$. In practise, the charge of the proton will be screened by electrons at atomic distance scales since the target will be made of hydrogen atoms.

d) The form factors, represented by G_1 and G_2, in the above equation take into account the fact that the proton is not a point-like particle, but that the charge and magnetic components are distributed in space. For point-like particles, $G_1 = G_2 = 1$. For the proton, though, the form factors are less than one since protons are not point-like particles. Argue that the two form factors will approach one in the case Q^2 approaches zero.

A very small Q^2 corresponds to a photon with a very long wavelength. If the wavelength is much larger than the size of the particle that you probe, the particle will appear as a point-like object, and hence, the form factors will become one.

e) In the case of deep-inelastic scattering (DIS), very large Q^2, the virtual photon will interact with one of the constituent quarks. To a good approximation, one could describe the process as (quasi-)elastic scattering of an electron on a single quark. Give an expression for the cross section in the case of DIS. Use the equation above as a starting point and modify the elements accordingly. Motivate your result.

In this case, G_1 and G_2 will be one, e.g. constant as a function of Q^2 (in essence Bjorken scaling). The fine structure constant α will be become $\alpha \times e_q$ whereby e_q the fraction charge of the quark ($2/3$ or $-1/3$), since the charge of the object will become fractional. The mass M will be replaced by the constituent mass of the quark, which is about $1/3$ of the proton mass. One has to sum over the contribution of all three constituent quarks, assuming that the process may be approximated by an incoherent sum.

This exam has been drafted by J.G. Messchendorp and verified by C.J.G. Onderwater.